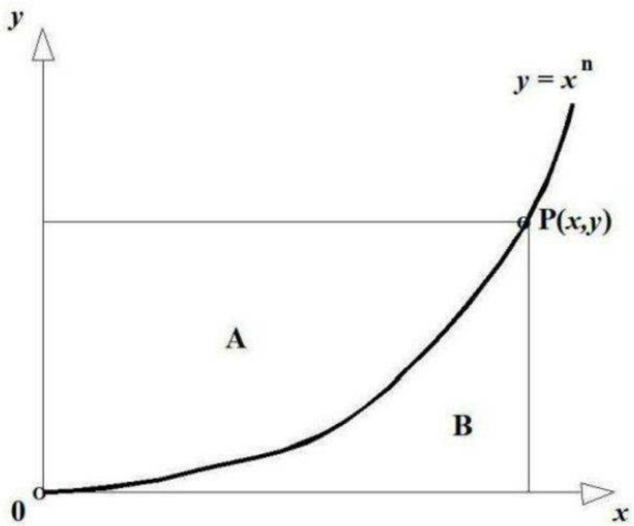
A new introduction to calculus for sixth-form students by Sidney Schuman

Calculus theory is generally introduced to students via a version of Newtonian fluxions. (Newton referred to a varying (flowing) quantity as a fluent and to its instantaneous rate of change as a fluxion.) The gradient of the curve $y = x^n$ is analysed in this way and the result is the differential power rule. The integral power rule is then shown (or more generally merely stated) as its inverse. I propose a *pre-calculus taster* based on area rather than gradient. Students can obtain the integral power rule as shown below and its inverse is the differential power rule.

Go straight to the integral power rule

First, a simple piece of algebra that produces an unexpected result.



Given that $\int x^n dx = \frac{x^{n+1}}{n+1}$ (constant of integration omitted)

then in the diagram, $B = \frac{x^{n+1}}{n+1}$ (Equation 1)

Also, $A + B = xy \quad \therefore A + B = x^{n+1} \quad \therefore A = x^{n+1} - B$

$\therefore A = x^{n+1} - \frac{x^{n+1}}{n+1} \quad \therefore A = n\frac{x^{n+1}}{n+1}$ (Equation 2)

Comparing equations 1 an 2, we see that $\frac{A}{B} = n$

Based on the same diagram with, for example, $\boldsymbol{x = 10}$ and $\boldsymbol{n =}$ **2, 3, 4, 5 …** students can use the mid-ordinate rule with **10** vertical strips to calculate area **B** for each value of $\boldsymbol{n}$.

Area **A** can then be calculated for each value of **B** (since $A + B = xy$) to obtain the ratio $\frac{A}{B} = n$. (See Note 1)

Students can then be encouraged to logically deduce the integral power rule as follows:

$\frac{A}{B} = n \quad \therefore A = Bn \quad$ Now, $A + B = xy \quad \therefore Bn + B = xy \quad \therefore B(n+1) = xy \quad \therefore B = \frac{xy}{n+1} \quad \therefore B = \frac{x^{n+1}}{n+1}$

Compare this result to the integral power rule as formally presented: $\int x^n dx = \frac{x^{n+1}}{n+1}$ (constant of integration omitted)

Calculus is a difficult part of the maths syllabus, causing anxiety and even fear among many sudents. My purpose in introducing a new approach is to relieve that anxiety. In my experience as a maths teacher, students understand the concept of area more easily than the concept of gradient. This pre-calculus taster allows them to become acquainted with a small part of calculus theory, leading to their more readily accepting classical instruction.